ECE 2117 — MODULE 1 CONCEPT CHECK REVIEWER

Logic and Mathematical Proofs
Identification-focused summary based on the uploaded Module 1 PDF.

1. Proposition & Predicate

• **Proposition** — a declarative sentence with exactly one truth value: True or False.

• **Atomic proposition** — a proposition not built from smaller propositions.

• **Predicate** — a sentence containing variables that becomes a proposition when values are assigned or variables are quantified.

• **Domain** — the set of values/objects over which a quantified statement is interpreted.

2. Logical Connectives

• **¬p — Negation:** NOT p.

• **p ∧ q — Conjunction:** p AND q; both must be true.

• **p ∨ q — Disjunction:** p OR q, including both.

• **p ⊕ q — Exclusive disjunction:** exactly one is true.

• **p → q — Conditional:** if p, then q.

• **p ↔ q — Biconditional:** p if and only if q.

3. Truth Tables & Classification

• **Conditional:** p → q is FALSE only when p is TRUE and q is FALSE.

• **Tautology:** true under every truth assignment — **ALWAYS TRUE**.

• **Contradiction:** false under every truth assignment — **ALWAYS FALSE**.

• **Contingency:** true for some assignments and false for others — **MIXED**.

• **Satisfiable:** true under at least one truth assignment.

• **Number of rows:** n variables → 2^n rows.

4. Conditional Terminology

• **Antecedent:** p in p → q.

• **Consequent:** q in p → q.

• **Converse:** q → p.

• **Inverse:** ¬p → ¬q.

• **Contrapositive:** ¬q → ¬p.

• **Key pair:** original ≡ contrapositive; converse ≡ inverse.

• **Important wording:** “p only if q” means p → q.

5. Logical Equivalence & Laws

• **Logical equivalence:** P ≡ Q means the formulas have the same truth value under every assignment.

• **Implication:** p → q ≡ ¬p ∨ q.

• **Negated implication:** ¬(p → q) ≡ p ∧ ¬q.

• **De Morgan:** ¬(p ∧ q) ≡ ¬p ∨ ¬q.

• **De Morgan:** $\neg(p \vee q) \equiv \neg p \wedge \neg q$.

• **Biconditional:** $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$.

• **Complement:** $p \vee \neg p \equiv T$; $p \wedge \neg p \equiv F$.

• **Double negation:** $\neg\neg p \equiv p$.

6. Quantifiers

• $\forall$ **— Universal quantifier:** for all / every.

• $\exists$ **— Existential quantifier:** there exists / at least one.

• $\exists!$ **— Unique existence:** exactly one.

• **Negation rule:** switch $\forall$ and $\exists$, then negate the predicate.

• $\neg\forall x\ P(x) \equiv \exists x\ \neg P(x)$.

• $\neg\exists x\ P(x) \equiv \forall x\ \neg P(x)$.

• **Quantifier order matters:** $\forall x \exists y$ can have y depend on x; $\exists y \forall x$ requires one fixed y to work for every x.

7. Arguments & Inference

• **Argument:** premises followed by a conclusion.

• **Valid argument:** no truth assignment makes all premises true and the conclusion false.

• **Sound argument:** valid argument + all premises are true.

• **Modus Ponens:** $p,\ p \rightarrow q\ \therefore\ q$.

• **Modus Tollens:** $p \rightarrow q,\ \neg q\ \therefore\ \neg p$.

• **Hypothetical Syllogism:** $p \rightarrow q,\ q \rightarrow r\ \therefore\ p \rightarrow r$.

• **Disjunctive Syllogism:** $p \vee q,\ \neg p\ \therefore\ q$.

• **Addition:** $p\ \therefore\ p \vee q$.

• **Simplification:** $p \wedge q\ \therefore\ p$.

• **Conjunction:** $p,\ q\ \therefore\ p \wedge q$.

8. Common Fallacies

• **Affirming the consequent:** $p \rightarrow q,\ q\ \therefore\ p$ — INVALID.

• **Denying the antecedent:** $p \rightarrow q,\ \neg p\ \therefore\ \neg q$ — INVALID.

9. Proof Methods

• **Direct proof:** assume the hypothesis and derive the conclusion.

• **Contraposition:** prove $\neg Q \rightarrow \neg P$ instead of $P \rightarrow Q$.

• **Contradiction:** assume the negation of the target and derive an impossibility.

• **Proof by cases:** divide the domain into exhaustive cases and prove the claim in each case.

• **Existence:** provide a witness and verify it satisfies the property.

• **Uniqueness:** assume two witnesses satisfy the property and prove they are equal.

• **Counterexample:** one valid example that disproves a universal claim.

10. Mathematical Induction

• **Base case:** prove the statement for the starting value.

• **Inductive hypothesis:** assume P(k) is true.

• **Inductive step:** use P(k) to prove P(k+1).

• **Strong induction:** the next case may use several or all previous cases.

• **Critical warning:** an induction step must prove P(k) → P(k+1); assuming P(k+1) is circular reasoning.

11. Identification Must-Know List

• **Always True → Tautology**

• **Always False → Contradiction**

• **Sometimes True/Sometimes False → Contingency**

• **True at least once → Satisfiable**

• **One example disproving a universal claim → Counterexample**

• **p in p → q → Antecedent**

• **q in p → q → Consequent**

• **Reverse → Converse**

• **Negate both → Inverse**

• **Reverse + negate → Contrapositive**

• **For all → Universal quantifier**

• **There exists → Existential quantifier**

• **Base case + inductive step → Mathematical induction**

• **Uses earlier cases → Strong induction**

LAST-MINUTE MEMORY: Tautology = ALL TRUE; Contradiction = ALL FALSE; Contingency = MIXED. Conditional fails only at T → F. Original ↔ Contrapositive. Converse ↔ Inverse. Negating quantifiers: switch ∀/∃ and negate the predicate.